

Worksheet 2 — Invertibility, inverses, and symmetry

Sections 1.5, 1.6, and 1.8 • Student worksheet

Learning goals. Recognize invertibility efficiently, use a supplied inverse, and write short proofs involving inverses, products, and transposes.

Name(s): _____

Section/group: _____

Core practice

1. Invertible or singular—without computing an inverse

Decide whether each matrix is invertible. Use elimination or a determinant argument, but do not compute an inverse.

$$A = \begin{bmatrix} -2 & 3 & -1 \\ 2 & 6 & 4 \\ 2 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 & 3 \\ -3 & 5 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

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2. Use an inverse as a computational tool

The inverse of A is supplied. Use it to solve $Ax = b$. Then check the result by multiplying A by your vector.

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & -3 \\ 2 & 1 & 10 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} -17 & -10 & -3 \\ -16 & -10 & -3 \\ 5 & 3 & 1 \end{bmatrix},$$
$$b = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$$

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3. Proof pair: inverse order and symmetric polynomials

Write a proof for each statement.

- (a) If A and B are invertible, then $(AB)^{-1} = B^{-1}A^{-1}$.
- (b) If A is symmetric, then $A^2 + 2A + 2I$ is symmetric.

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Extension / challenge

4. Extension: transpose and inverse

Prove that if A is invertible, then A^T is invertible and $(A^T)^{-1} = (A^{-1})^T$.

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