

Worksheet 11 — Symmetric eigenvalues and singular values

Sections 8.5 and 8.7 • Student worksheet

Learning goals. Use the spectral structure of symmetric matrices, test whether proposed eigen-data are possible, and compute singular values from $A^T A$ or AA^T .

Name(s): _____

Section/group: _____

Core practice

1. Positive definite implies positive eigenvalues

Let K be a real symmetric positive-definite matrix. Prove that every eigenvalue of K is positive. Your proof should start from an eigenpair and use the defining quadratic-form inequality.

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2. Can these eigen-data come from a symmetric matrix?

For each proposed set of eigen-data for a real 2×2 symmetric matrix, decide whether it is possible. If possible, identify the matrix; if impossible, cite the violated theorem.

- (a) Eigenvalue 2 for both independent vectors $(2,1)^T$ and $(1,2)^T$.
- (b) Eigenvalue 3 for $(2,-1)^T$ and eigenvalue -1 for $(-1,2)^T$.

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3. Singular values of a square and a wide matrix

Compute all nonzero singular values of each matrix. For the 2×4 matrix, choose the smaller of $A^T A$ and AA^T to minimize work.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 0 & -1 & 1 & 1 \end{bmatrix}.$$

Extension / challenge

4. Extension: singular-value true/false

Decide whether each statement is true. Prove a true statement and give a counterexample to a false one.

- (a) A and A^T have the same nonzero singular values.
- (b) If A is upper triangular with positive diagonal entries, then its singular values are exactly those diagonal entries.

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