

Worksheet 1 — Matrix algebra, elimination, and factorization

Sections 1.1–1.4 • Student worksheet

Learning goals. Use matrix laws carefully, solve systems by elimination, and interpret an elimination factorization as a sequence of simpler solves.

Name(s): _____

Section/group: _____

Core practice

1. Matrix statements: always, sometimes, or never?

For conformable real matrices, classify each statement. Prove every statement you mark *always*; give a concrete counterexample for every statement you mark *sometimes* or *never*.

(a) $A + (B + C) = (A + B) + C$. (b) $AB = BA$. (c) $AB = 0$ implies $A = 0$ or $B = 0$. (d) $A(B + C) = AB + AC$.

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2. One system, two solution routes

Solve the system first by Gaussian elimination. Then factor its coefficient matrix as $A = LU$ and solve again through $Lc = b$ followed by $Ux = c$.

$$\begin{aligned} 3x_1 - 5x_2 &= 7, \\ -6x_1 + 8x_2 &= 2. \end{aligned}$$

Explain how the elimination multiplier appears in L .

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3. A factorization with a row exchange

For the matrix and right-hand side below: (a) find a permutation matrix P and factors L, D, V such that $PA = LDV$, where L and V have diagonal entries 1; (b) use the factorization to solve $Ax = b$.

$$A = \begin{bmatrix} 0 & 1 & -3 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

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Extension / challenge

4. Extension: a determinant forecast

Without expanding by cofactors, use your factorization from Problem 3 to determine $\det(A)$. Explain the contribution from the row exchange.

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