# Research Statement

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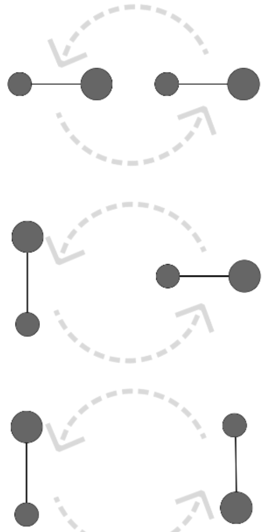
In the urban landscape where I grew up, the city lights obscured much of the sky. Nonetheless, I found clear nights in which to pierce the haze with my little telescope, and found a fascination in staring at the stars and planets; speculating about the inner workings of the universe. Now I research celestial gravity in two of its manifestations: orbital mechanics and black hole gravitational waves.

As it relates to orbital mechanics, I wrote my dissertation on the two-body problem. That is, the problem of predicting the motions of two bodies that are interacting freely due to gravity (imagine two asteroids out in space). Newton solved a simplified version of the problem, where one assumes that each body is merely a point mass. However, the full two-body problem (where we do not make such an assumption) is still an open problem. Since this describes a dynamical system, our first task is to identify equilibria of the system (configurations where the bodies can remain motionless). However, because the bodies are orbiting each other, we don't expect a proper equilibrium solution. But we might be able to find "relative equilibria" (RE), where the bodies are moving (orbiting each other), but they maintain a constant radius, and they do not rotate relative to each other. Another task is to determine whether these RE are stable, that is, whether such a configuration, when perturbed, will remain close to the RE.

In my research I approximate the full two-body problem by modeling each body not as a point mass (as Newton did), but by two point-masses connected with a massless rod, a "dumbbell." With this model, I revealed symmetric RE (configurations in which the two bodies are colinear, perpendicular, or parallel), as well as some asymmetric RE bifurcating from these symmetrical ones. I've also located an upper bound on the radius of these asymmetrical RE and characterized their stability (both linear and nonlinear).

*Dumbbells Rotating*

The mathematics involved includes generating a Lagrangian from the kinetic and potential energies of the configuration, and then using the Euler-Lagrange equation to find the equations of motion. I also reduce the equations by using the fact that at a RE there is rotational symmetry. Another researcher, Smale, showed that after such a reduction, the RE can then be found as critical points of the reduced potential. As a bonus, if these critical points are minima, Smale showed that they are nonlinearly stable.



*Symmetric RE*

But why do we care? Practically speaking, many of the finite resources humans use on Earth can be found in asteroids, many of which are irregular in shape. The shapes of these asteroids and the probes we'll be sending there can be modeled by masses connected by rods. And the calculations a spacecraft will need to make to find a static orbit or RE around these asteroids will need to be based on gravitational models that consider their irregular shapes. Finding these RE is the aim of my research.

I was lucky as an undergraduate to have conducted research into environmental science with my mathematical modeling professor, Rikki Wagstrom. Knowing of my interest in celestial objects, upon my graduation she suggested I join a research team working on black holes. Since then, I have conducted research with mathematician Michael Green and physicist Ramin Daghigh at Metropolitan State University, St. Paul.

Einstein taught us that when the universe gives birth to a black hole, it cries out with gravity waves. My colleagues and I calculate the "sound" that different types of black holes make at birth, or when otherwise perturbed. A black hole's gravity waves are analogous to an earthquake's seismographic waves, they can be read as curvy lines on a page (called waveforms). A few years ago, scientists were finally able to detect gravitational waves in this way. But what can we learn about black holes from looking at these waveforms? Can we determine the type or size of the black hole?

My colleagues and I take mathematical models (partial differential equations) representing black holes of different types, and then calculate the gravity waves predicted by these models. As the models grow in complexity, they gravity waves they produce should match those found by the gravity wave detectors. That way, when scientists detect gravity waves from space, they can compare those waveforms to those calculated from this type of research, and thereby determine the type of black hole which generated the waves. Additionally, since there is still much debate about the exact structure of black holes, researchers can compare the waveforms predicted by competing theories to determine which theory is correct.

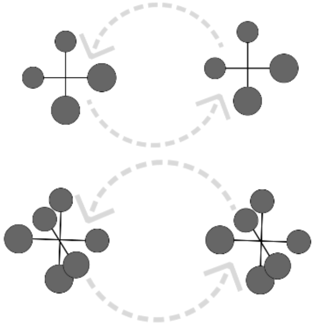
Specifically, my colleagues and I recently published a paper in Physical Review D that examines the quasinormal modes (QNMs) produced by a "regular" black hole. QNMs relationship to waveforms is analogous to the relationship individual notes have to a musical chord. QNMs are indivisible components which, once added together, produce the waveform. A *regular* black hole is one for which there is no singularity. Instead, the central mass is compact, but finite. In our research, the QNMs were calculated to have negative imaginary parts, this implies damping (the amplitude of the waves will diminish over time). This further implies the black hole will not gravitationally radiate away all its mass, i.e., it is stable.

We also published a paper in Physical Review D which introduced a technique to simplify the calculations involved in the process of finding the waveforms. Some mathematical models lend themselves more easily to waveform calculation. For the more difficult models, we showed you can approximate the gravitational potential appearing in the differential equation with one allowing for more easy calculations, while still approximating the waveform to any accuracy desired.

The mathematics involved in calculating QNMs includes substituting an educated guess for the solution (an "ansatz," which turns out to be a series), into the differential equation. Subsequently, we find from this equation an n-term recurrence relation, we turn this relation into a new equation involving an infinite continued fraction, and then numerically solve that equation for the QNMs. While this process was originally done for the simplest of black holes, my contribution has been to generalize this process for implementation with more complicated theoretical models. These generalizations are coded using Mathematica.

## Future Research

I have plans to expand my two-body problem research to include bodies consisting of 2 dumbbells perpendicular to each other, connected along their rods, a "pinwheel." This way, each body is two-dimensional, instead of the one-dimensional dumbbell. The end goal would be eventually to model each body as a three-dimensional object, with 3 dumbbells connected along their rods, a "jack." This model would provide a lot of flexibility to approximate real-world objects (as you modify each body's six mass parameters, and three rod length parameters).



*Pinwheels and Jacks*

Regarding gravitational waves, my colleagues and I are planning to study other theoretical models for "regular" black holes, possibly using the approximation technique described in our earlier paper to produce gravitational waveforms and checking for stability. Results from this research will help to reveal which theoretical black hole models represent real black holes, and which models can be eliminated.

I am lucky to have been able to pursue my childhood fascination. Not only have I had the chance to learn about those motions in the night sky, but also to contribute to the scientific conversation. I also look forward to providing undergraduates with a similar opportunity to the ones I was afforded. There are many extensions of my two-body problem work that an undergraduate could pursue with my guidance. In addition, some of the research I did as an undergraduate into automobile fossil fuel consumption and electoral voting systems could be updated and extended by an undergraduate researcher. I look forward to making these opportunities available, as well as continuing my own research path.