MATH 1271: Calculus I

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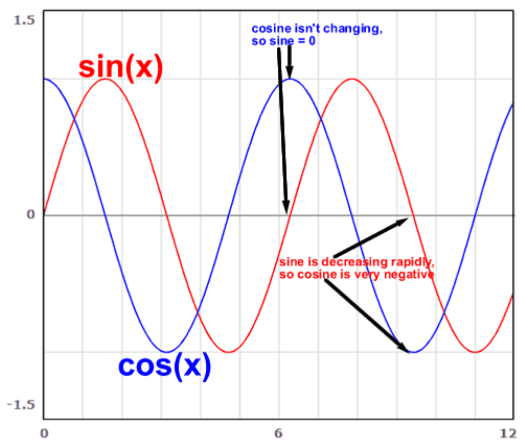
3.3 - Derivatives of Trigonometric Functions

Review



Recall that csc $= \frac{1}{\sin}$, sec $= \frac{1}{\cos}$, and cot $= \frac{1}{\tan} = \frac{\cos}{\sin}$.

Using the limit definition of a derivative, it is known that:

$$(\sin x)'|_{x=0} = \lim_{x\to 0} \frac{\sin(x-0)-\sin 0}{x} = \lim_{x\to 0} \frac{\sin x}{x} = 1,$$

and $(\cos x)|'_{x=0} = \lim_{x\to 0} \frac{\cos x - 1}{x} = 0.$

So this must mean that at $x = 0$, we have $(\sin x)' = 1 = \cos x$, and $(\cos x)' = 0 = -\sin x$.

More generally for all x we have:

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\tan x) = \sec^2 x$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\frac{d}{dx}(\cot x) = -\csc^2 x$

Problem 3. Differentiate $f(x) = \sin x + \frac{1}{2} \cot x$.

$$f(x) = \sin x + \frac{1}{2} \frac{\cos x}{\sin x}$$

$$f' = \cos x + \frac{1}{2} \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$$

$$f' = \cos x + \frac{1}{2}\,\frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$f' = \cos x - \frac{1}{2}\,\frac{1}{\sin^2 x}$$

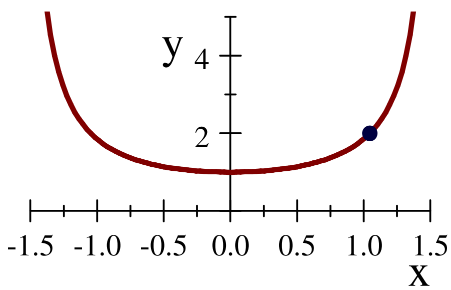
$$f' = \cos x - \frac{1}{2}\csc^2 x$$ [or just memorize the part of the review for $(\cot x)'$]

Problem 9. Differentiate $y = \frac{x}{2-\tan x}$.

$$y' = \frac{(1)(2-\tan x) - x(-\sec^2 x)}{(2-\tan x)^2}$$

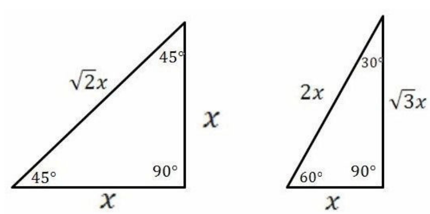
$$= \frac{2-\tan x + x\sec^2 x}{(2-\tan x)^2}.$$

Problem 21. Find an equation of the tangent line to $y = \sec x$ at the point $(\frac{\pi}{3}, 2)$



$$y' = \sec x \tan x.$$

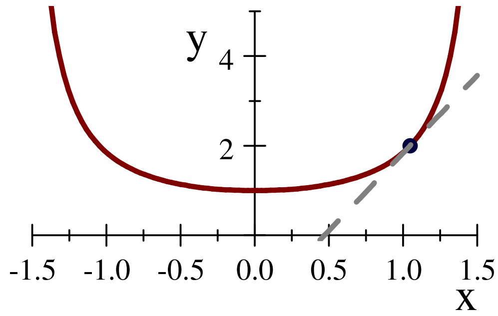
So $y'(\frac{\pi}{3}) = \sec\frac{\pi}{3}\tan\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}}\,\frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}$



$$= \frac{1}{\frac{1}{2}}\,\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}.$$

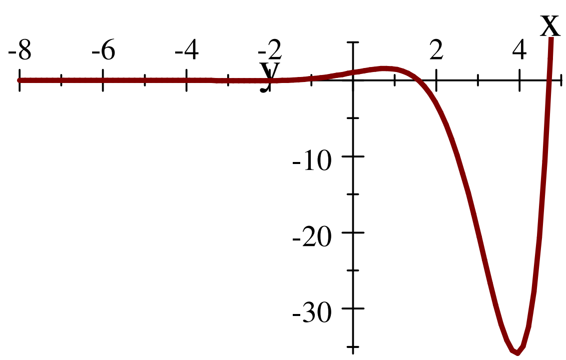
Equation of the tangent line:

$y - 2 = 2\sqrt{3}\,(x - \frac{\pi}{3})$ or $y = 2\sqrt{3}\,x + 2 - \frac{2\pi}{3}\sqrt{3}$.



$\sec x$ and $2\sqrt{3}\,x + 2 - \frac{2\sqrt{3}\,\pi}{3}$

Problem 34. For what values of x does the graph of $f = e^x \cos x$ have a horizontal tangent?



$f(x) = e^x \cos x$ has a horizontal tangent when $f' = 0$.

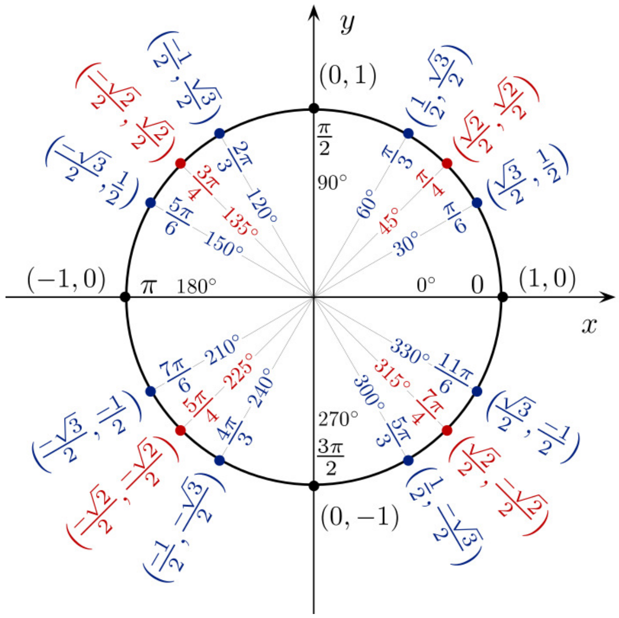
$f' = e^x(\cos x) + e^x(-\sin x)$

$= e^x(\cos x - \sin x)$.

$f' = 0 \quad \Rightarrow \quad \cos x - \sin x = 0$

$\Rightarrow \cos x = \sin x$

$\Rightarrow x = \frac{\pi}{4} + n\pi,$ where n an integer.



Problem 43. Find $\lim\limits_{x\to 0} \frac{\sin 3x}{5x^3-4x}$.

Since we already know that $\lim\limits_{3x\to 0} \frac{\sin 3x}{3x} = 1$, we would prefer to solve something in the form: $\lim\limits_{3x\to 0} [\frac{\sin 3x}{3x} \cdot g(x)]$, for some mathematical expression $g(x)$.

So we want $\frac{\sin 3x}{5x^3-4x} = \frac{\sin 3x}{3x} \cdot g(x)$.

Solving for $g(x)$, we find: $g(x) = \frac{3x \sin 3x}{5x^3-4x} = \frac{3}{5x^2-4}$.

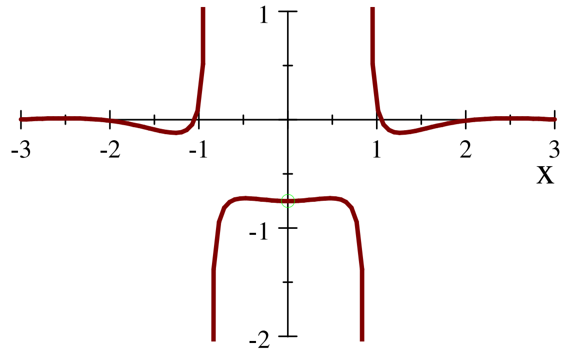
So, $\lim\limits_{x\to 0} \frac{\sin 3x}{5x^3-4x} =$

$$= \lim_{x\to 0} \left(\frac{\sin 3x}{3x} \cdot \frac{3}{5x^2-4} \right)$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{3}{5x^2-4}$$

Where in the first limit, we did a change of variable $x \to \frac{1}{3}x$.

$$= 1 \cdot (\frac{3}{-4}) = -\frac{3}{4}.$$



$$\frac{\sin 3x}{5x^3-4x}$$