MATH 1271: Calculus I

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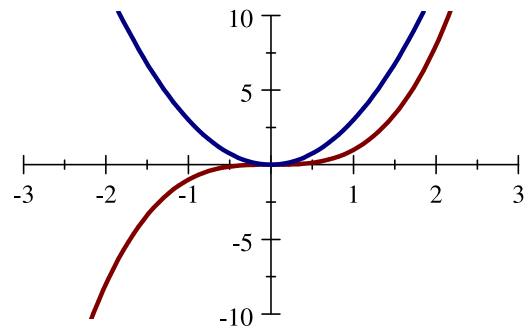
2.8 - Derivative as a Function

Review:

f' **as a Function**: Given any point x at which the derivative $f'(x)$ exists, we assign to x the number $f'(x)$ (the slope of f at x). In this way, we regard f' as a function in-and-of itself, and we call the function f', **the derivative of** f.

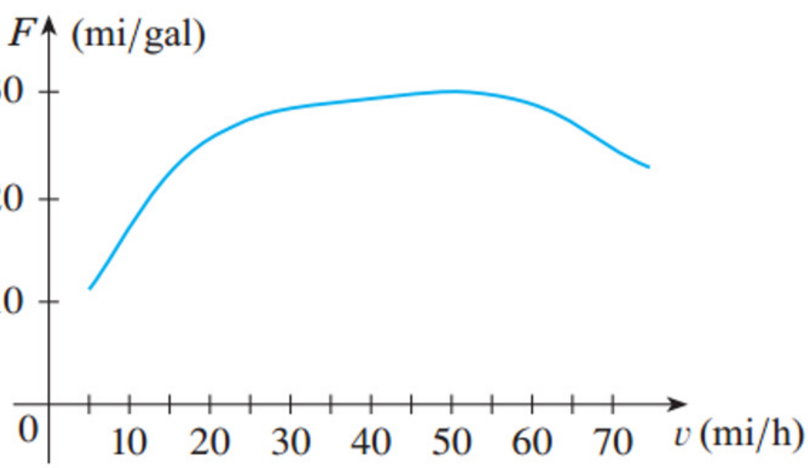


$f = x^3$ and $f' = 3x^2$

Definition: A function f is **differentiable at** a if $f'(a)$ exists. We say that f is **differentiable on an open interval** (a, b) [or (a, ∞) or $(-\infty, a)$ or $(-\infty, \infty)$] if it is differentiable at every point in the interval.

Differentiability Implies Continuity: If f is differentiable at a, then f is continuous at a.

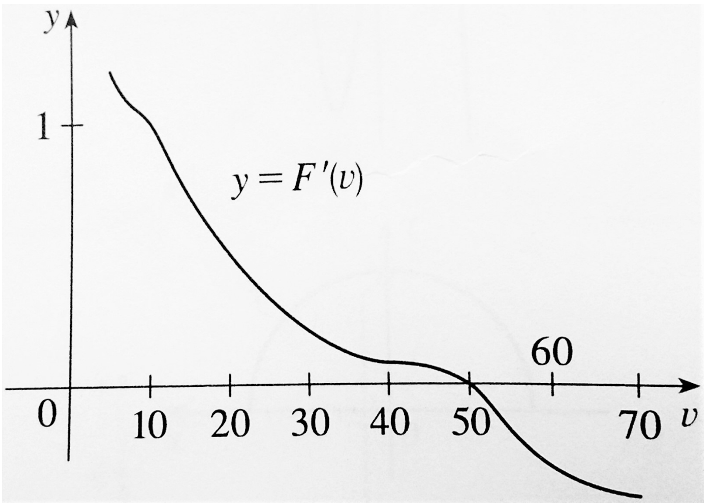
Problem 14. The graph below (from the US Department of Energy) shows how driving speed affects gas mileage. Fuel economy F is measured in miles per gallon and speed v is measured in miles per hour.



a. What is the meaning of the derivative $F'(v)$?

$F'(v)$ is the instantaneous rate of change of fuel economy with respect to speed.

b. Sketch the graph of $F'(v)$.



c. At what speed should you drive if you want to save on gas?

Problem 21. Find the derivative of $f(x) = \frac{1}{2}x - \frac{1}{3}$ using the definition of derivative. State the domain of the function and the domain of its derivative.

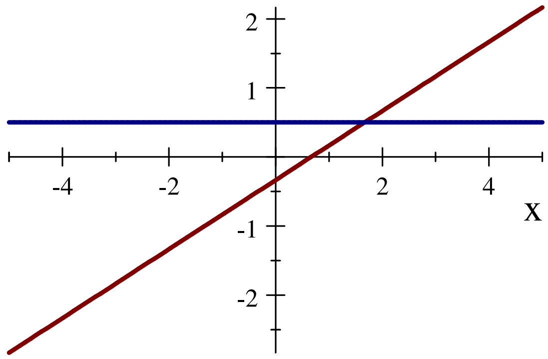
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left[\frac{1}{2}(x+h)-\frac{1}{3}\right]-\left(\frac{1}{2}x-\frac{1}{3}\right)}{h} = \lim_{h\to 0} \frac{\frac{1}{2}x+\frac{1}{2}h-\frac{1}{3}-\frac{1}{2}x+\frac{1}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}h}{h} = \lim_{h\to 0} \frac{1}{2} = \frac{1}{2}.$$

Domain of function $\frac{1}{2}x - \frac{1}{3}$?

Domain of derivative?



$$f = \frac{1}{2}x - \frac{1}{3},\quad f' = \frac{1}{2}$$

Problem 26. Find the derivative of $g(t) = \frac{1}{\sqrt{t}}$ using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f' = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} \frac{\sqrt{t}\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}$$

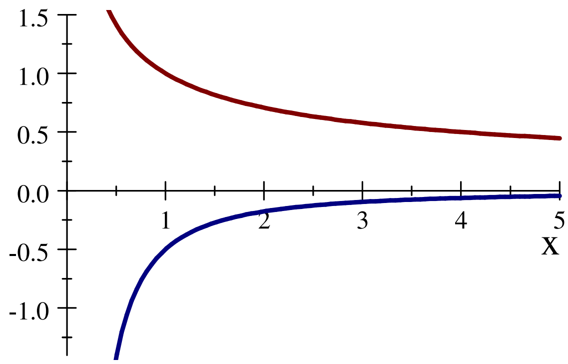
$$= \lim_{h\to 0} \frac{\sqrt{t}-\sqrt{t+h}}{h\sqrt{t}\sqrt{t+h}}$$

$$= \lim_{h\to 0} \frac{\sqrt{t}-\sqrt{t+h}}{h\sqrt{t}\sqrt{t+h}} \frac{\sqrt{t}+\sqrt{t+h}}{\sqrt{t}+\sqrt{t+h}} = \lim_{h\to 0} \frac{t-(t+h)}{h\sqrt{t}\sqrt{t+h}\left(\sqrt{t}+\sqrt{t+h}\right)} = \lim_{h\to 0} \frac{-h}{h\sqrt{t}\sqrt{t+h}\left(\sqrt{t}+\sqrt{t+h}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{t}\sqrt{t+h}\left(\sqrt{t}+\sqrt{t+h}\right)} = \frac{-1}{\sqrt{t}\sqrt{t}(\sqrt{t}+\sqrt{t})} = \frac{-1}{2t\sqrt{t}}$$

Domain of function $\frac{1}{\sqrt{t}}$?

Domain of derivative?



$f = \frac{1}{\sqrt{t}}, \quad f' = \frac{-1}{2t\sqrt{t}}$

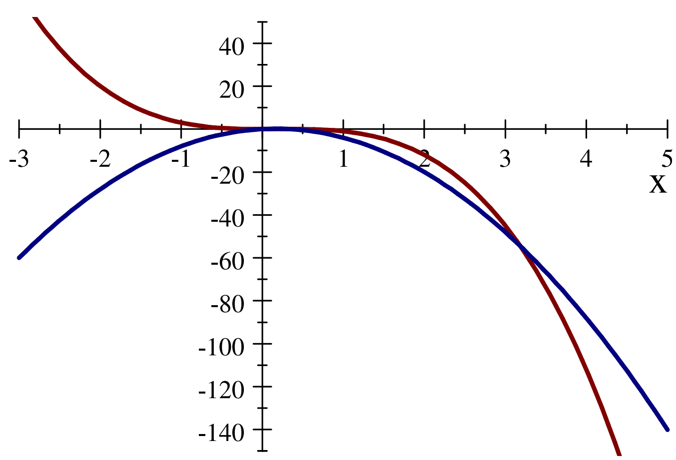
Problem 25. Find the derivative of $f(x) = x^2 - 2x^3$ using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left[(x+h)^2-2(x+h)^3\right]-(x^2-2x^3)}{h}$$

$$= \lim_{h\to 0} \frac{(x^2+2xh+h^2)+(-2x^3-6x^2h-6xh^2-2h^3)-(x^2-2x^3)}{h} = \lim_{h\to 0} \frac{2xh+h^2-6x^2h-6xh^2-2h^3}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h-6x^2-6xh-2h^2)}{h} = \lim_{h\to 0} (2x+h-6x^2-6xh-2h^2) = 2x-6x^2.$$

Domain of function $x^2 - 2x^3$?
Domain of derivative?



$f = x^2 - 2x^3, \quad f' = 2x - 6x^2$