MATH 1272: Calculus II

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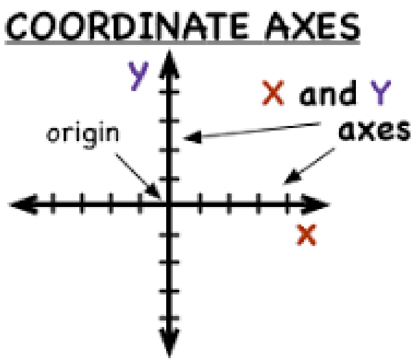
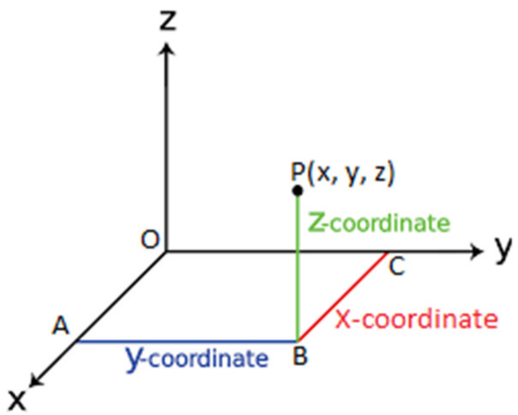
12.1 - Three-Dimensional Coordinate Systems

Review:

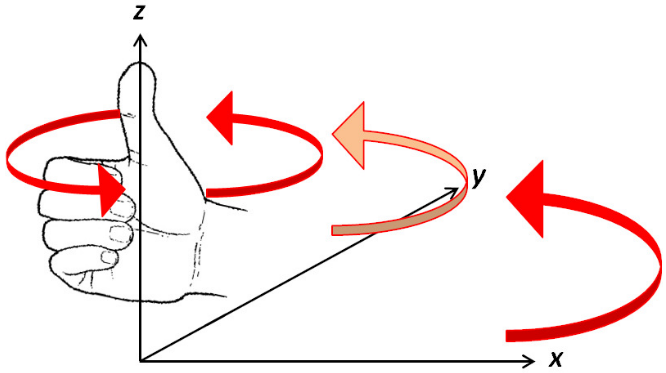


Coordinate Axes: Directed lines (for example, the x-axis, y-axis, z-axis) through the origin which are perpendicular to each other.

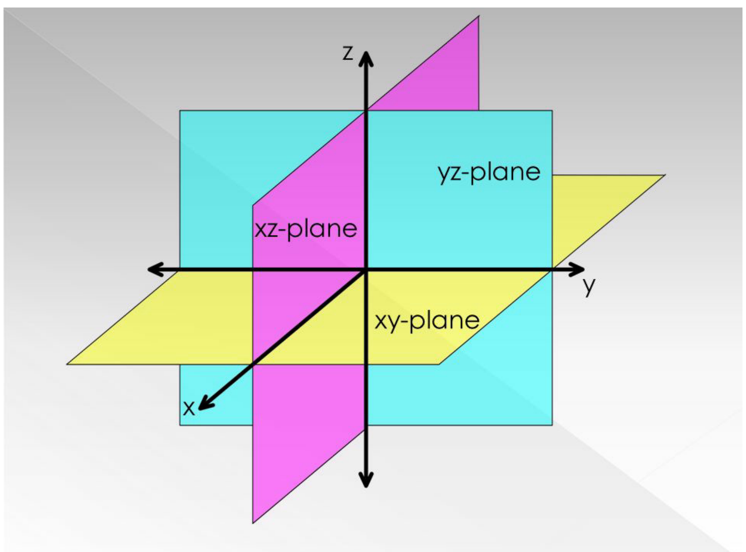
Coordinates: If we represent a point P in three-dimensional space as the **ordered triplet** (a,b,c), where a is the distance along the x-axis, b is the distance along the y-axis, and c is the distance along the z-axis; we call a,b,c the coordinates of P.



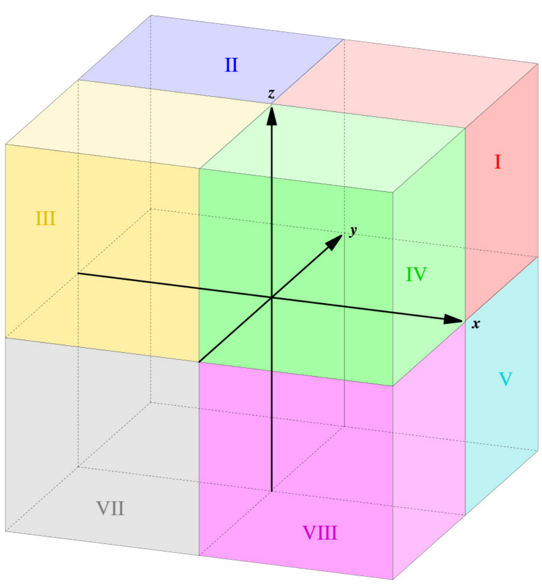
Right-Hand Rule: If you imagine curling the fingers of your right hand around the z-axis in the direction of a 90° counterclockwise rotation, starting from the positive x-axis, and continuing to the positive y-axis, then your thumb ends up pointing in the direction of the positive z-axis.



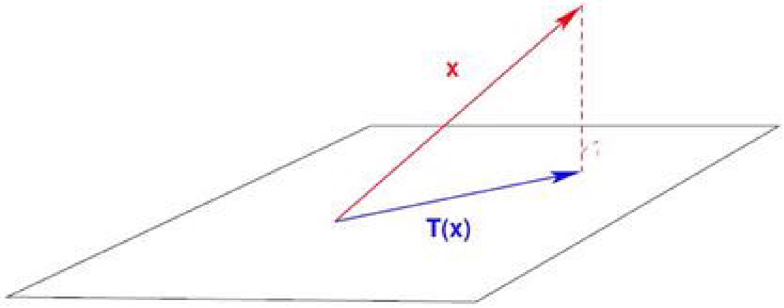
Coordinate Planes: If you take two of the axes (for example, x, y), and take all of the points either on or directly between the two axes, you end up with a coordinate plane (for example, the xy-plane).



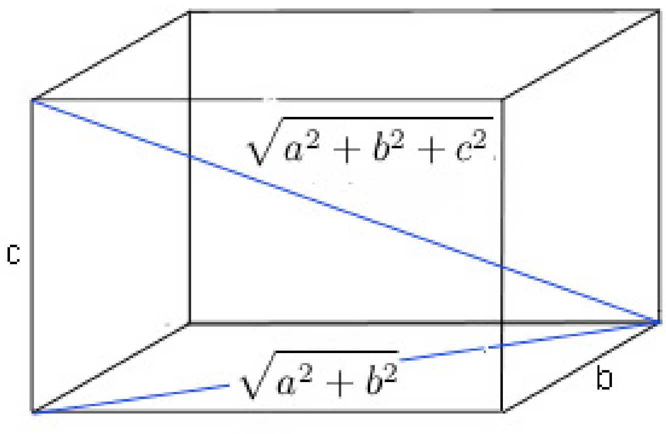
Octants: The coordinate planes divide three-dimensional space into eight areas, these areas are called octants, and the octant with positive entries for all three coordinates is called the **first octant**.



Projection: For any point (a,b,c) in three-dimensional space, we call $(a,b,0)$ a projection of P on to the xy-plane, we call $(a,0,c)$ a projectiononto the xz-plane, and so on.



Distance Formula in Three Dimensions: The distance $|P_1P_2|$ between the points $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$ is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.



Equation of a Sphere: An equation of a sphere with center $C(h,k,l)$ and radius r is $(x-h)^2+(y-k)^2+(z-l)^2 = r^2$.

Problem #8 Let $P(2,-1,0)$, $Q(4,1,1)$, $R(4,-5,4)$ be the vertices of a triangle PQR. Find the lengths of the sides of the triangle. Is it a right triangle? Is it an isosceles triangle?

$$|PQ| = \sqrt{(q_1-p_1)^2+(q_2-p_2)^2+(q_3-p_3)^2}$$
$$= \sqrt{(4-2)^2+(1-(-1))^2+(1-0)^2} = \sqrt{4+4+1} = 3.$$

$$|QR| = \sqrt{(4-4)^2+(1+5)^2+(1-4)^2} = \sqrt{0+36+9} = 3\sqrt{5} \approx 6.7$$

$$|RP| = \sqrt{(4-2)^2+(-5+1)^2+(4-0)^2} = \sqrt{4+16+16} = 6.$$

Obviously not isosceles. Right triangle?

Does the sum of the squares of the shorter sides equal the square of the large side?
$3^2+6^2 = 9+36 = 45 = 9 \cdot 5 = \left(3\sqrt{5}\right)^2$ ✓

Problem #18 Show that the equation $3x^2+3y^2+3z^2 = 10+6y+12z$ represents a sphere, and find its center and radius.

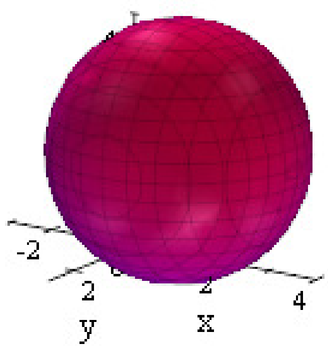
$$3x^2+3y^2-6y+3z^2-12z = 10$$

$x^2 + y^2 - 2y + z^2 - 4z = \frac{10}{3}$

$x^2 + (y^2 - 2y + 1) + (z^2 - 4z + 2) = \frac{10}{3} + 1 + 2$

$x^2 + (y - 1)^2 + (z - 2)^2 = \frac{19}{3}$

Center is $(0, 1, 2)$, and radius is $\sqrt{\frac{19}{3}}$.



$3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z$

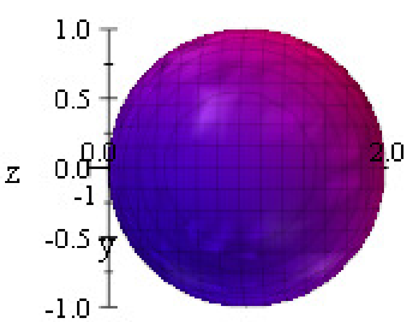
Problem #34 Describe in words the region of $\mathbb{R}^3$ represented by the inequality $x^2 + y^2 + z^2 > 2x$.

$x^2 - 2x + y^2 + z^2 > 0$

$(x^2 - 2x + 1) + y^2 + z^2 > 1$

$(x - 1)^2 + y^2 + z^2 > 1$

Therefore, $x^2 + y^2 + z^2 > 2x$ represents the region of $\mathbb{R}^3$ that is exterior to the sphere centered at $(1, 0, 0)$ with a radius 1.



$(x - 1)^2 + y^2 + z^2 = 1$

Problem #38 Write inequalies to describe a solid upper hemisphere of a sphere of radius 2 centered at the origin.

$x^2 + y^2 + z^2 \leq 4,$ and $z \geq 0.$