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7.1 - Integration by Parts

Review:

The most important integrals you should have run across so far…

$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ (n \neq -1)$ $\int \frac{1}{x} dx = \ln|x| + C$

$\int e^x dx = e^x + C$ $\int a^x dx = \frac{a^x}{\ln a} + C$

$\int \sin x\ dx = -\cos x + C$ $\int \cos x\ dx = \sin x + C$

$\int \sec^2 x\ dx = \tan x + C$ $\int \csc^2 x\ dx = -\cot x + C$

$\int \sec x \tan x\ dx = \sec x + C$ $\int \csc x \cot x\ dx = -\csc x + C$

$\int \sinh x\ dx = \cosh x + C$ $\int \cosh x\ dx = \sinh x + C$

$\int \tan x\ dx = \ln|\sec x| + C$ $\int \cot x\ dx = \ln|\sin x| + C$

$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}(\frac{x}{a}) + C$ $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(\frac{x}{a}) + C, \quad a > 0$

Integration by Parts (w/ indefinite integral):

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx \qquad (1)$$

OR

$$\int u\ dv = uv - \int v\ du \qquad (2)$$

Integration by Parts (w/ definite integral):

$$\int_a^b f(x)g'(x)dx = f(x)g(x)]_a^b - \int_a^b g(x)f'(x)dx$$

Problem #6 Evaluate $\int (x-1)\sin(\pi x)dx$.

Let $u = x-1$, and $dv = \sin(\pi x)dx$.

So, $du = dx$, and $v = -\frac{1}{\pi}\cos(\pi x)$.

Then by Equation 2 above, $\int (x-1)\sin(\pi x)dx = -\frac{1}{\pi}(x-1)\cos \pi x - \int(-\frac{1}{\pi}\cos(\pi x)dx)$

$= -\frac{1}{\pi}(x-1)\cos(\pi x) + \frac{1}{\pi}\int \cos(\pi x)dx$

$= -\frac{1}{\pi}(x-1)\cos(\pi x) + \frac{1}{\pi^2}\sin(\pi x) + C.$

Problem #36 Evaluate $\int_0^t e^s \sin(t-s)ds$.

Let $u = \sin(t-s)$, and $dv = e^s ds$.

So, $du = -\cos(t-s)ds$, and $v = e^s$.

Then $I = \int_0^t e^s \sin(t-s)ds = [e^s \sin(t-s)]_0^t + \int_0^t e^s \cos(t-s)ds$

$= e^t \sin 0 - e^0 \sin t + I_1 = -\sin t + I_1.$

For I_1, let $U = \cos(t-s)$ and $dV = e^s ds$.

So, $dU = \sin(t-s)ds$, and $V = e^s$.

So $I_1 = [e^s \cos(t-s)]_0^t - \int_0^t e^s \sin(t-s)ds = e^t \cos 0 - e^0 \cos t - I$
$= e^t - \cos t - I.$

Thus, $I = -\sin t + I_1 = -\sin t + e^t - \cos t - I.$

Or, $2I = e^t - \cos t - \sin t \quad \Rightarrow \quad I = \frac{1}{2}(e^t - \cos t - \sin t).$

Problem #40 First make a substitution and then use integration by parts to evaluate $\int_0^\pi e^{\cos t} \sin 2t dt$.

Let $x = \cos t$, so that $dx = -\sin t dt$.

And $\sin 2t = 2 \sin t \cos t$.

Thus, $\int_0^\pi e^{\cos t} \sin 2t dt = -\int_{t=0}^{t=\pi} e^x \frac{\sin 2t}{\sin t} dx$

$= -\int_1^{-1} e^x \frac{2 \sin t \cos t}{\sin t} dx$

$= 2\int_{-1}^1 xe^x dx.$

Now use IBPs with $u = x,\ dv = e^x dx$, which gives us $du = dx,\ v = e^x$, so we get:

$2\int_{-1}^1 xe^x dx = 2\left([xe^x]_{-1}^1 - \int_{-1}^1 e^x dx\right)$

$= 2\left(e^1 + e^{-1} - [e^x]_{-1}^1\right)$

$= 2(e + \frac{1}{e} - [e - \frac{1}{e}]) = 2(\frac{2}{e}) = \frac{4}{e.}$

Problem #52 Use integration by parts to prove the following reduction formula:

$$\int x^n e^x dx = x^n e^x - n\int x^{n-1} e^x dx.$$

Let $u = x^n$ and $dv = e^x dx$.

This implies $du = nx^{n-1} dx$, and $v = e^x$.

By equation 2 we have, $\int x^n e^x dx = x^n e^x - n\int x^{n-1} e^x dx.$